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The control of mixing by manipulation of instability modes leading to the formation of vortical structures has a direct impact on the performance of propulsion systems. In the plane mixing layer, mixing is accomplished by two-dimensional entrainment associated with spanwise vortices, and three-dimensional motion induced by packets of streamwise counter-rotating vortex pairs. Our research goal is to advance the state of understanding of the basic fluid mechanics of the mixing layer to aid in the implementation of real-time closed loop control schemes. To this end, the evolution of spanwise and streamwise instabilities has been investigated by independent forcing in the streamwise and spanwise directions. The flow is forced by means of a mosaic of individually controlled surface heaters, which allows for flexible programming of complex spatial/temporal forms of excitation. The downstream evolution of the spanwise instability and its dependence on the configuration are studied using Schlieren visualization and velocity measurements taken with a rake of hot wire probes. Pulsed 2-D and 3-D forcing is also used to study the temporal evolution of the flow.

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I. INTRODUCTION AND OVERVIEW

The study of the mechanisms which lead to mixing transition in plane and axisymmetric mixing layers owes much of its importance to technological applications in combustion (Oppenheim 1987, Decker and Schafer 1988). These processes involve chemical reaction between two or more species within a free turbulent shear flow, and depend crucially on mixing of the flow streams. Thus, the development of methods for the control of mixing through manipulation of the flow will have a direct impact on the performance of propulsion systems from the standpoint of efficient combustion and controllable thrust.

Turbulent chemically reacting flows present a formidable task for the development of schemes to control their evolution because of the intimate coupling between the chemical, thermal, and fluid mechanical phenomena. It appears that when diffusion at the Kolmogorov microscale is fast, as is often the case in high Reynolds number combustion systems, efficient mixing of the chemical species is limited by fluid motions induced by a hierarchy of large coherent vortical structures. In the plane shear layer, mixing is accomplished by nominally two-dimensional entrainment of irrotational fluid from both streams by the spanwise vortices, and three-dimensional motion induced by packets of streamwise counter-rotating vortex pairs. These vortices evolve from two- and three-dimensional instabilities of the mean flow and hence may be manipulated by small-amplitude acoustic, mechanical, or thermal excitation at the flow partition, with dramatic effects on the global properties of the mean flow such as cross-stream spreading and reaction rates. For combustion processes, the excitation may influence the overall reaction rate, as well as the spatial and temporal distribution of heat release and reaction products.

In real-time control applications of nominally two-dimensional shear layer flows of practical interest, it is a nontrivial problem that these flows are not only irregular both in time and space, but are also subjected to temporally and spatially complex disturbances with important consequences to the mixing. We have begun to address these issues in an experimental investigation of a plane turbulent mixing layer subjected to pulsed excitation.

Complex spatial and temporal manipulation of the plane mixing layer require that the excitation be applied using actuators having fast dynamic response. Moreover, the actuator(s) should be capable of affecting the dynamics of the large coherent structures at low excitation amplitude. The effectiveness of fully two-dimensional arrays of surface-mounted heating elements in manipulating the two- and three-dimensional vortical structures in a water shear layer facility has been convincingly demonstrated in our experiments. An added advantage of these actuators is their

dynamic reconfigurability, which is achieved by independent control of each element by the laboratory computer.

In §II, we review details of previous work related to mixing transition in plane shear layers. In §III, we report on our progress to date. Publications acknowledging this AFOSR Grant are listed in §IV.

II. TECHNICAL BACKGROUND

The development of schemes for the manipulation and control of chemically reacting turbulent flows presents a formidable task because of the intimate coupling between the chemical, thermal, and fluid mechanical phenomena. As discussed in the Introduction, efficient mixing of the reactants is limited by fluid motions driven by the large coherent vortical structures. These vortices, which originate in the two- and three-dimensional (2-D and 3-D) instabilities of the mean flow, can be manipulated by small-amplitude excitation (e.g., mecahnical or, heating inputs) at the flow partition, with dramatic effects on the global properties of the mean flow, such as cross-stream spreading (Oster and Wygnanski 1982) and reaction rates (Roberts and Roshko 1985). For combustion processes, an important consequence of the forcing is the potential for controlled spatial enhancement or reduction of mixing, including the possibility of redistribution of the reactants, products or heat release within the shear layer. In this section we review the state-of-the art in experimental and theoretical fluid mechanics relevant to control of mixing in the plane shear layer.

The amount of reaction product formed in the mixing layer between two reacting streams increases by an order of magnitude through the mixing transition region downstream from the flow partition (Roshko, 1981). The three-dimensional motion necessary for the enhancement of mixing is associated with the appearance of streamwise counter-rotating vortex pairs first observed in plan view by Miksad (1972) and Brown and Roshko (1974). Lin and Corcos (1984) demonstrated, in a time-dependent three-dimensional numerical simulation, that the streamwise vorticity appearing in a two-dimensional shear layer increases the mixing rate between the two streams due to the growth of the material interfaces.

Flow visualization snapshots of a chemically reacting shear layer in a water tunnel (Breidenthal, 1978) revealed the existence of a spanwise sinuous wiggle around the primary vortex. The spanwise wavelength of this wiggle is approximately the same as the streamwise wavelength of the initial two-dimensional instability. The amplitude of the wiggle grows rapidly as it is convected downstream, apparently as a result of stretching by the adjacent primary vortices, with no appreciable change in its wavelength. Time exposure photographs of the plan view show streaks that appear at the onset of the wiggle, have almost the same spanwise wavelength (approximately that of the local vorticity thickness), and extend through the end of the test section.

Time exposure photographs were also obtained by Konrad (1976) and Bernal and Roshko (1986) in a gas mixing layer facility. The streaks were observed over a large range of Reynolds numbers and their mean onset Reynolds number was found to increase as the shear layer velocity

ratio increased. The mean spanwise spacing of the streaks scaled with the vorticity thickness at their onset and increased somewhat with downstream distance. Similar observations were reported by Browand and Trout (1980) and Miksad (1972). Hot wire measurements by Jimenez (1983) yielded contours of streamwise mean velocity which were plotted as a function of the spanwise and streamwise coordinates at a fixed cross-stream elevation. These plots closely resemble the streamwise streaks in the time exposure photographs. <u>It is quite remarkable that the spatial coherence and spanwise wavelength of the streamwise vortices are preserved despite the intense interactions between the primary (spanwise) vortices.</u>

A model of the three-dimensional structure in a plane mixing layer was first introduced by Bernal (1981). Based on a flow visualization study and high-speed cinephotography, Bernal concluded that the counter-rotating streamwise vortex pairs comprise part cf a vortex that continuously loops back and forth between adjacent primary (spanwise) vortices. The passage frequency of these corrugated vortices is locked to that of the primary vortices and their spanwise wavelength tends to increase somewhat as they are convected downstream. However, the increase in the spanwise wavelength is much less than the change in scale of the primary structure. More recently, Lasheras et al. (1986) experimentally studied the initial development of the three-dimensional instability which leads to the formation of the streamwise vortices. They showed that the onset of these vortices can be moved considerably downstream by careful removal of upstream disturbances from the flow. They also demonstrated that streamwise vortices may be easily triggered by small vortex generating elements mounted on the flow partition or, as reported in a later paper (Lasheras & Choi, 1987), by using a corrugated splitter plate.

There is a considerable similarity between the appearance of the spanwise sinuous wiggle around the primary vortices in a plane shear layer prior to mixing transition, and the azimuthal core instability of an isolated vortex ring (Glezer, 1987). In an experimental study of a turbulent vortex ring, Glezer and Coles (1987) concluded that vortex tubes, alternating in sense, wrap around the core of the main vortex and induce flow corresponding to entrainment or de-entrainment in certain azimuthal planes. Photographs by Didden (1977) clearly show the formation of such vortex tubes upon the transition to turbulence of the main vortex. These observations are of particular interest because they indicate that vortex-ring transition may be similar to the mixing transition in the plane mixing layer. Conversely, the fact that counter-rotating vortex pairs appear around the core of an isolated vortex ring in connection with the azimuthal core instability suggests that a spanwise core instability of the two-dimensional vortices in a plane mixing layer may be coupled with the appearance of the streamwise vortices.

The evolution of the large scale two-dimensional motion in a plane mixing layer results from an instability of the mean flow and may be viewed as the conglomeration of interacting instability waves that propagate and amplify in the streamwise direction (Ho & Huerre, 1984). Pierrehumbert and Widnall (1982) identified another instability mode in a shear layer described by an array of Stewart vortices. This instability is in the spanwise direction and repeats in the streamwise direction with the frequency of the two-dimensional flow. There is an indication that this convective instability can lead to the generation of streamwise vorticity. The numerical calculations of Lin and Corcos (1984) showed that an initially weak distribution of streamwise vorticity in a uniform straining flow (as may exist between two consecutive spanwise vortices) may evolve into concentrated round streamwise vortices. The evolution of such streamwise vortices was studied numerically by Ashurst and Meiburg (1985) who pointed out the sensitivity of the flow to upstream variations in the spanwise vorticity distribution. The numerical results of Metcalfe et al. (1985) show that spanwise instability modes are convected with the flow, grow at rates similar to those of the two-dimensional modes, and evolve into pairs of counter-rotating streamwise vortices in the braids between the primary vortices. These results also indicate that persistent pairing of the primary vortices may inhibit the three-dimensional instability, while suppression of pairing of the primary vortices may drive the three-dimensional modes to turbulent-like states.

Although very little is known about the interaction between the spanwise and streamwise instability modes, the literature on forced mixing layers suggests some possible links. The application of low-level temporally harmonic excitation to free shear flows has been widely accepted as an important diagnostic tool for the study of individual instability modes present within the broad disturbance spectrum associated with noise in real flows (Ho & Huerre, 1984). Moreover, the use of harmonic spanwise uniform forcing has provided a powerful tool for the manipulation of some stability modes with dramatic effects on the global properties of the flow. Oster and Wygnanski (1982) discovered that forcing a two-dimensional turbulent shear layer produces a "frequency locked" region in which the growth is inhibited, the primary vortices are equally spaced, and their passage frequency is equal to the forcing frequency. They also reported that the intensity of spanwise velocity fluctuations in this region diminishes and the flow becomes more two-dimensional. Roberts and Roshko (1985) reported that when the "frequency locked region" occurred downstream of the mixing transition, forcing could either enhance or reduce the degree of mixing, and correspondingly alter the amount of reaction product. If one assumes that, at sufficiently high Reynolds numbers, mixing is connected with three-dimensional motion induced by the appearance

of streamwise vortices, these observations suggest that two-dimensional forcing may have a considerable effect on the three-dimensional structure of the flow.

Experimental evidence suggests that the streamwise vortices tend to lock to small geometric imperfections which either exist within the experimental apparatus (Bernal, 1981) or are artificially introduced on the flow partition (Jimenez, 1983). Lasheras and Choi (1987) demonstrated that the plane mixing layer is extremely sensitive to upstream spanwise nonuniformities in the vorticity distribution. Hence, we conclude that it is relatively easy to trigger an arbitrary spanwise distribution of streamwise vortices. This view is further supported by the work of Cohen (1985), who showed that within a given range of streamwise perturbations in the axisymmetric jet, all azimuthal modes amplify at an identical rate provided the jet diameter is much larger than the momentum thickness of the axisymmetric shear layer.

There is some evidence in the literature which suggests that the spanwise instability can evolve and amplify in the absence of the 2-D vortices. In a theoretical study of the nonlinear interaction between 2-D and 3-D waves in a plane shear layer Benney (1961) showed, without including the the 2-D vortices, that the spanwise instability evolves into streamwise counter-rotating vortices. Similarly, in a more recent numerical investigation of the evolution of solitary waves on a single spanwise vortex filament in a plane shear layer, Aref and Flinchem (1983) showed that the filament may develop spanwise undulations resembling the sinuous wiggle found by Breidenthal.

Based on our recent experiments (described in §III.2), we conclude that the 3-D instability in a plane shear layer results from the interaction of initial nonuniformities in the spanwise vorticity and the 2-D Kelvin-Helmholtz waves. Subsequent formation of spanwise vortices significantly alters the strain field and induces the appearance of spanwise packets of counter-rotating vortex pairs between adjacent spanwise vortices. These packets are convected with the flow and are continuously stretched and ingested by the spanwise vortices. Since the entrainment rate is determined by the dynamics of these vortices, mixing transition occurs when 3-D motion is established within their cores (at high enough Re). The fact that time exposure photographs show little streamwise variation of the spanwise wavelength suggests that the latter is fixed at the upstream onset of the instability and is convected with the flow.

III. FINAL TECHNICAL REPORT

III.1. Facility, Actuators, and Experimental Techniques

In this section, we briefly describe our water shear layer facility, the mosaic of flush mounted heating actuators, and the accompanying large suite of flow diagnostic tools that we have developed under the sponsorship of AFOSR and NSF. We believe that this hardware has been a key ingredient in the experimental findings described in §III.2.

III.1.a. The Water Shear Layer Facility

The facility is shown in Figure 1. The two streams originate from the same reservoir and are driven by a single pump powered by a 10 hp motor equipped with a solid state speed controller (VF PACK-P). The velocity of each stream and the velocity ratio can be independently varied. Test-section velocities up to 200 cm/sec can be realized. An additional test section and contraction have been recently acquired. The test section (100 cm long and either 10 cm x 22 cm span or 22 cm x 22 cm span) is equipped with independently removable Lucite walls so that the flow may be observed from any direction. The convergence of the test section on either side of the shear layer may be easily adjusted in order to vary the streamwise pressure gradient. The contraction (with a contraction ratio of either 7:1 or 9:1) has a rectangular cross section with a constant aspect ratio. The contraction contours were formed using a fifth-order polynomial, which minimizes the boundary layer thickness on the flow partition and side walls. Turning vanes and "turbulence manipulators" (honeycomb and screens) upstream of the contraction reduce velocity variations due to secondary flow. The trailing edge of the flow partition is replaceable and is configured with various mosaics of surface heaters. These actuators will be discussed at greater length in §III.1.c.

The quality of the flow in this facility has been extensively documented and reported in a previous AFOSR Annual Progress Report (1987).

III.1.b. Instrumentation

The facility is equipped with a suite of diagnostic instrumentation. The pressure transducer (Rosemount model 2051, 0.1% accuracy @ 2" H_2O) is connected to two 12-port fast switches (Scanivalve type W0601/1P-12T). These switches are computer controlled and allow for monitoring the velocity on either side of the contraction exit plane and the static pressure along the test section, as well as pitot-static measurements of the velocity field within the test section. The water temperature is monitored and recorded by the laboratory computer *via* a digital thermometer

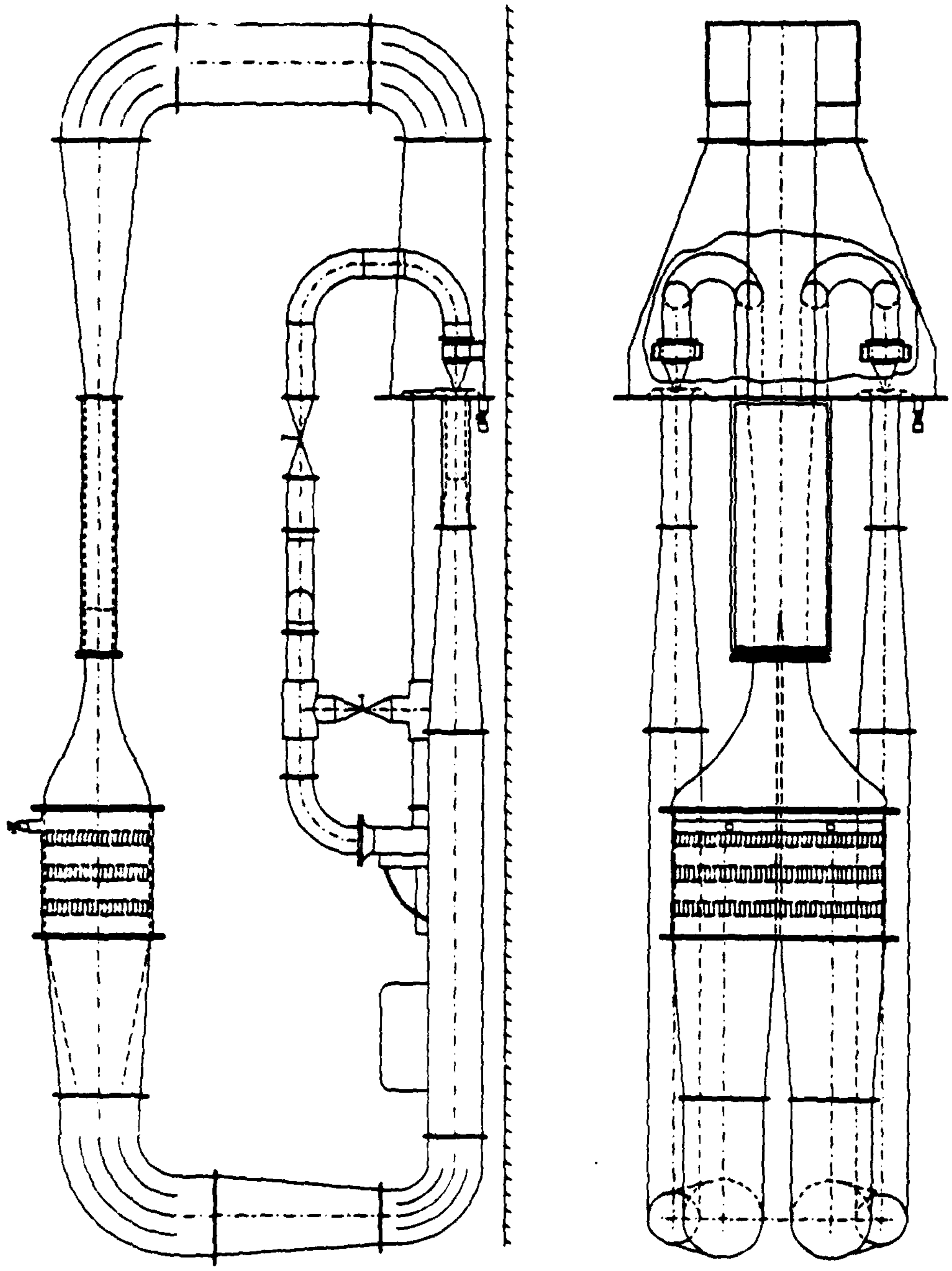


Figure 1. Sketch of water shear layer facility.

(Fluke model 2180A resolution 0.1° C). Eight dye injection ports are available on each side of the flow partition. A computer controlled two-axis traverse mechanism, designed for detailed measurements of the flow field within the test section with rakes of hot-wire probes, has been installed in the facility.

Twenty channels of hot-wire/film anemometry (AA Lab Systems) are now available for simultaneous measurement of instantaneous velocity distributions. A rake of 31 closely spaced hot-wire probes, suitable for use in water, has been developed and mounted on the test section traverse mechanism for simultaneous cross-stream or spanwise measurements of the streamwise velocity (Figure 2).

Detailed measurements at points of particular interest in the flow field can also be obtained by a two-component, frequency-biased laser Doppler velocimeter, the construction of which is nearing completion. Beam splitting and frequency shifting are accomplished by two partially overlapping radial phase gratings driven by hysteresis-synchronous motors. The three-watt Argon-ion laser (Lexel model 95) and its associated optics are already on hand. A three-channel digital processor has been designed and fabricated to convert the raw Doppler signals into a form suitable for input to the data acquisition system.

A Masscomp laboratory computer system, including 16 channels of 12 bit A/D, 16 channels of D/A, and 32 channels of general-purpose I/O, is dedicated to experiment control and data processing.

The introduction of a controlled vorticity distribution into the boundary layer of the flow partition by the surface heating actuators is accompanied by small localized density gradients in the adjacent fluid. The corresponding refractive index gradients are exploited for flow visualization by means of a sensitive Schlieren system designed and built for this purpose (Fiedler et al., 1985). This technique allows for a nonintrusive study of the effect of forcing on the flow in planes parallel and normal to the flow span. The resulting streaklines of heated fluid particles are recorded by a video camera in a form suitable for digital image processing.

A Vicom VME digital image processing system and two MTI-Dage (model 70 DX) video cameras are available for recording and processing of various full-field data from flow visualization (e.g., Schlieren, particle tracking and liquid crystal dye). This stand-alone system, acquired through a DOD University Research Instrumentation Program award, is based on a Sun Microsystems 3/160 computer and will enable the display, storage and processing of full-field data from the flow. The

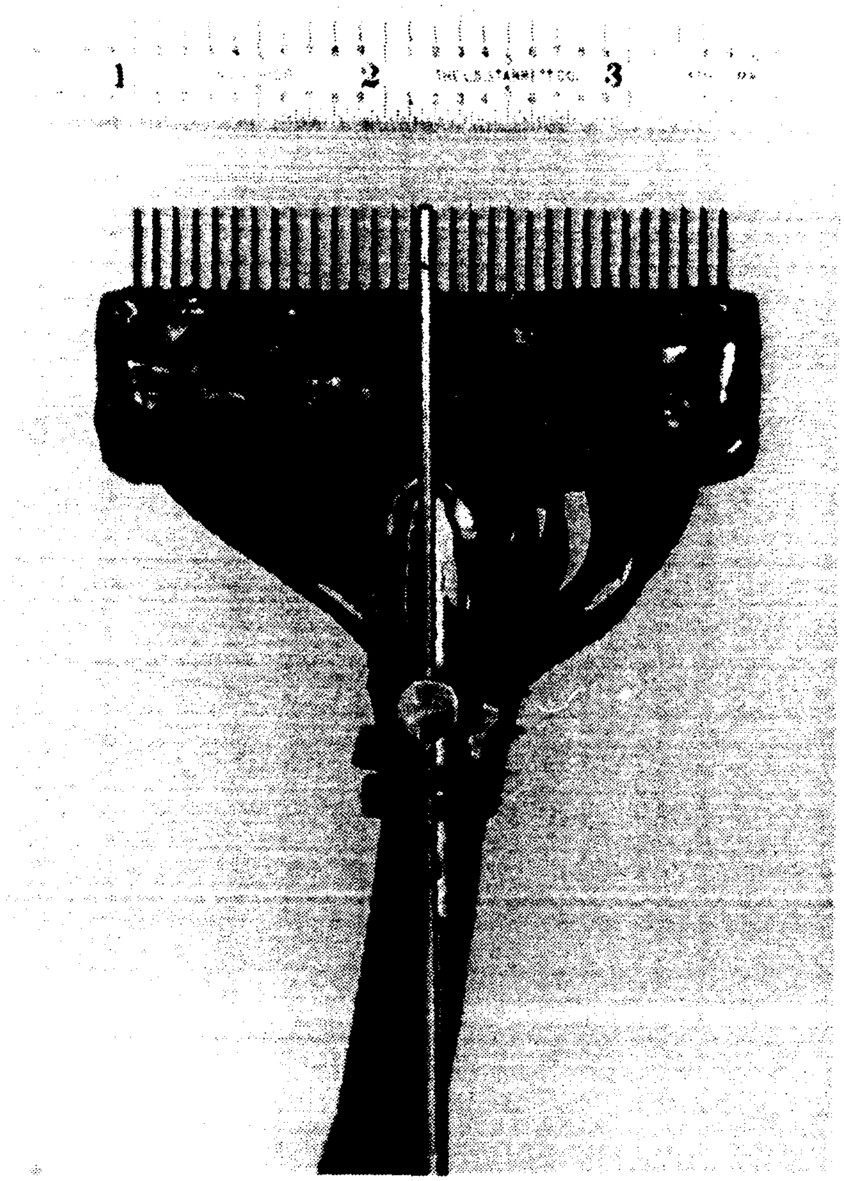


Figure 2. Hot wire rake.

system is able to perform high-speed frame masking, averaging, integration, and convolution on the incoming data.

III.1.c. Actuators: Surface Film Heaters

Excitation of streamwise and spanwise instability modes is accomplished in our water shear layer facility by a mosaic of surface film heaters mounted on the flow partition. The effect of heating the surface is essentially to introduce a controlled vorticity distribution into the flow partition's boundary layer (Liepmann et al., 1982). It is important to recognize that small induced oscillations in the boundary layer amplify or decay according to linear stability theory. Thus, forcing a shear layer from an upstream boundary layer may not be effective if the induced waves decay appreciably before reaching the trailing edge. Hence the forcing frequency should be within the unstable (amplified) range of the boundary layer, the extent of which depends strongly on the pressure gradient. By carefully extending the flow partition into the test section, we are able to tune the streamwise pressure gradient so that it becomes slightly adverse, thus causing the flow partition boundary layer to become less stable, and hence more receptive to forcing.

After considerable development, we have fabricated two mosaics of surface film heaters. Mosaic I consists of 14 spanwise-uniform elements, and two spanwise rows each having 16 individual elements. Mosaic II is comprised of four spanwise-uniform heaters upstream of a single spanwise row having 32 individual elements. The Inconel heating elements are mounted on a standard epoxy board substrate. A thin film coating, selected for good heat conduction, provides corrosion protection and electrical insulation. Each heating element is wired through the epoxy board (using through-hole plating) and the flow partition to an individual DC power amplifier. Thirty-two channels of power amplifiers, each capable of continuously driving 10 A into a load of 2-4 ohms are available. The unit's output is limited to 2.5 kW by the power supply. Sixteen channels of power amplifiers can be directly driven by the laboratory computer *via* a D/A interface. This allows input of arbitrary temporal waveforms to the heaters without distortion, by compensating in software for thermal variability of heater resistance, and for the quadratic dependence of Joulean dissipation on input voltage.

III.2. Results from Open-Loop Forcing of the Plane Mixing Layer

The purposes of our current AFOSR-supported investigation of a spanwise nonuniformly forced plane mixing layer are twofold. First, we are using open-loop excitation as a diagnostic tool for the detailed study of the origin and evolution of the spanwise (3-D) instability and the mechanism of its interaction with the streamwise (2-D) instability. Second, we are using the

temporal and spatial flexibility of our recently developed surface film heaters to establish the ability of these actuators to manipulate the large coherent structures which control the mixing.

As discussed in this section, we have convincingly demonstrated the effectiveness of our surface film actuators to exert control authority over the development and evolution of the large vortical structures in the plane mixing layer.

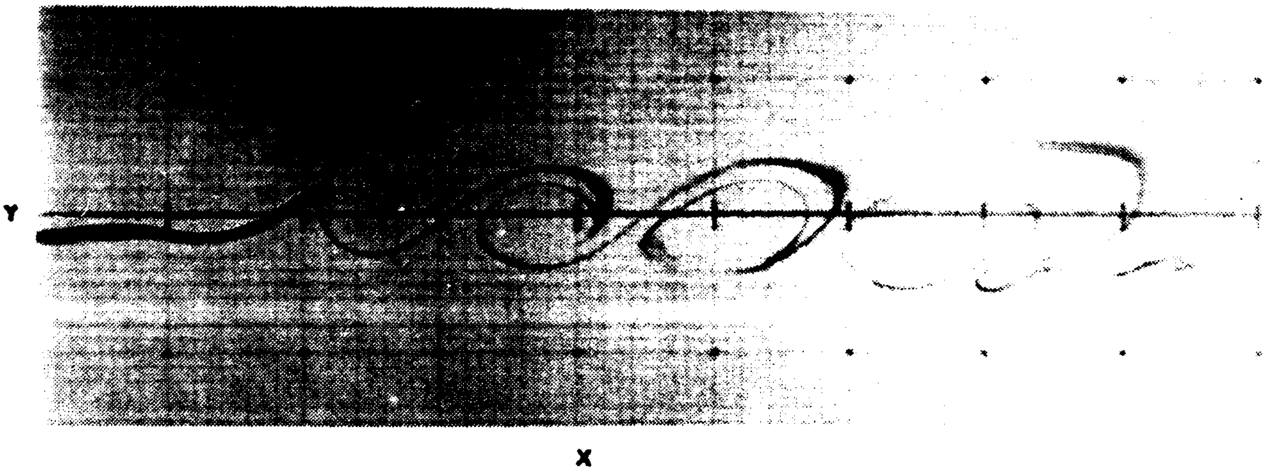
III.2.a. Two-dimensional Excitation by Heating Actuators

Photographs in the cross-stream plane of a mixing layer subjected to spanwise uniform excitation are shown in Figure 3. The flow is visualized by dye injected into the boundary layer of the low-speed side. When the forcing frequency is equal to the natural frequency (f_0) of the shear layer (Figure 3a), the spanwise vortices appear to be formed at f_0 and initially move downstream in an orderly fashion without amalgamation. When the forcing frequency is equal to $f_{0/2}$ (Figure 3b), vortices are shed from the trailing edge of the flow partition at f_0 and immediately undergo pairing to form a larger vortex. These results are in agreement with previous observations of Ho and Huang (1982) and Roberts (1985) in low-speed water facilities.

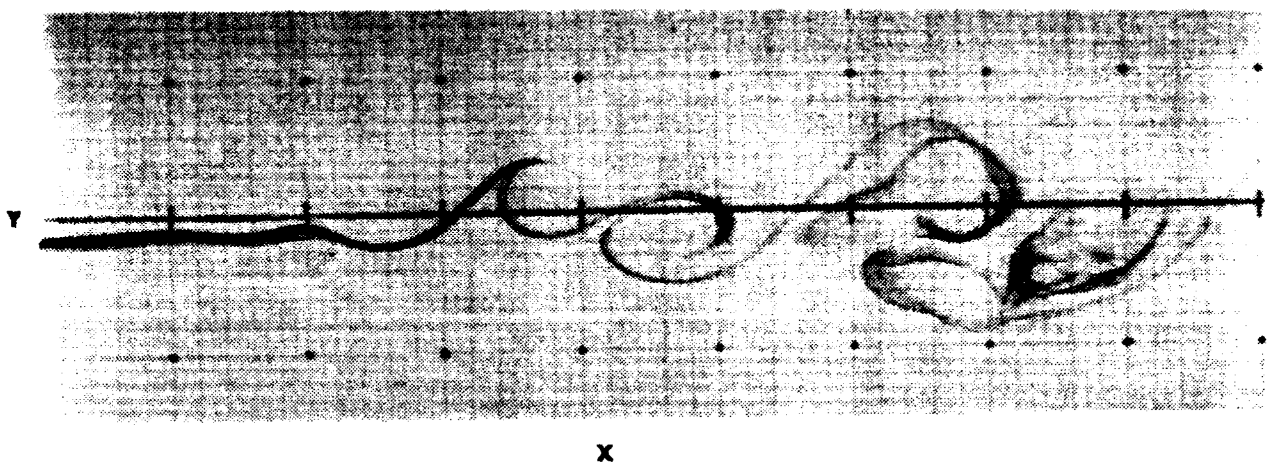
Cross-stream profiles of mean streamwise velocity of the forced flow measured at a number of streamwise locations are plotted in similarity coordinates in Figure 4. Corresponding velocity profiles of the unforced flow are also shown for comparison. These data clearly demonstrate that the forced shear layer spreads more in the cross-stream direction than does the unforced flow. Although the velocity profiles of the forced flow appear to be self-similar, the difference in momentum thickness ($\Delta\Theta$ in Figure 5) between the corresponding forced and unforced flows shows considerable streamwise variation. This is reminiscent of the downstream variation of momentum thickness in the experiments of Ho and Huang (1982) under similar forcing conditions.

III.2.b. Three-dimensional Excitation by Heating Actuators

In the plane shear layer, mixing is accomplished by nominally 2-D entrainment of irrotational fluid from both streams by the spanwise vortices, and 3-D motion induced by packets of streamwise counter-rotating vortex pairs which form in the region of maximum strain between adjacent spanwise vortices (the "braids"). Although it is widely accepted that initial spanwise nonuniformity of the vorticity in a plane mixing layer leads to the evolution of streamwise vortices, very little is known about the mechanism by which the 3-D instability spreads in the span.



(a)



(b)

Figure 3. Photographs in the cross-stream plane of a mixing layer subjected to two-dimensional excitation. a) at the natural frequency (6 Hz); b) at one-half the natural frequency (3 Hz).

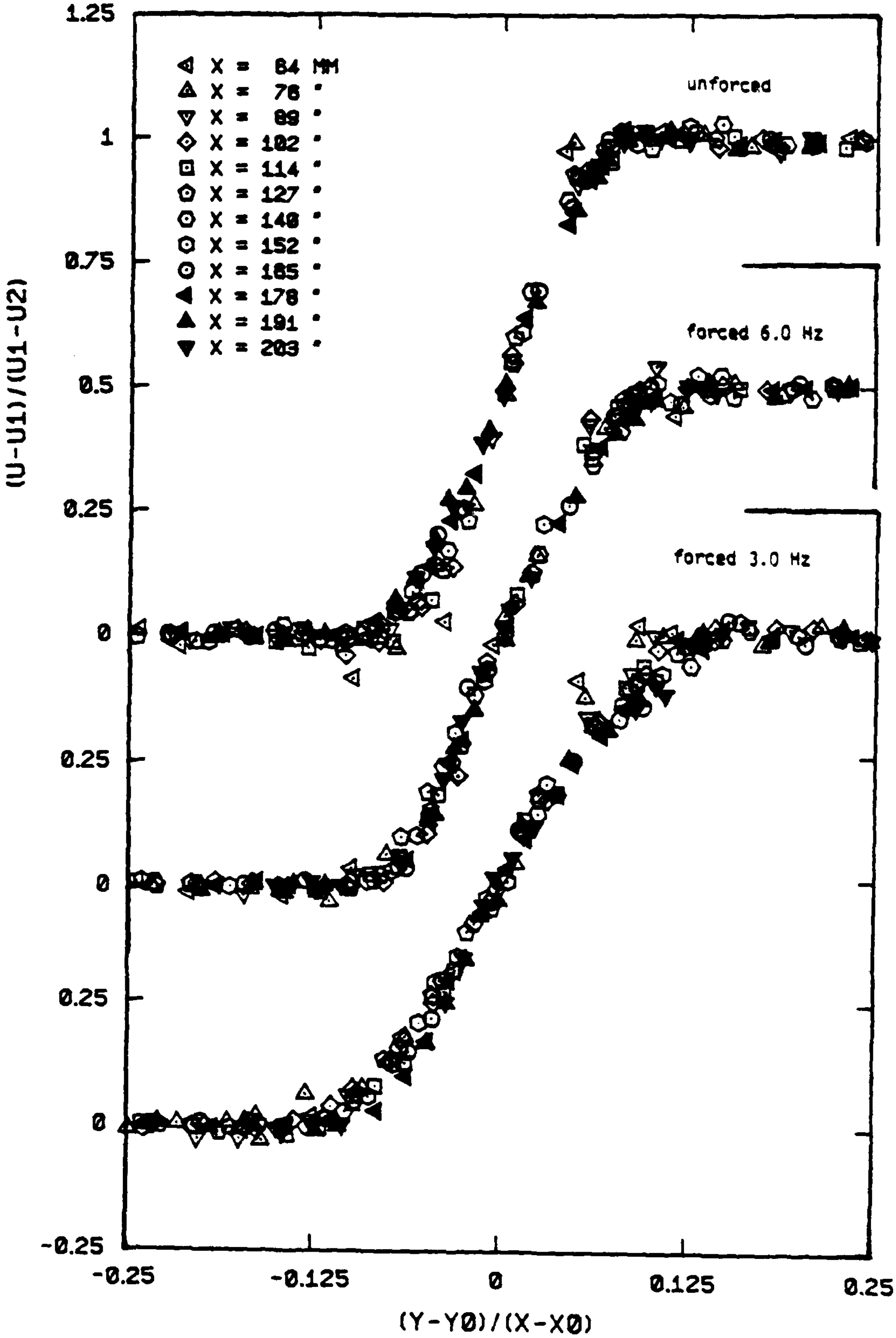


Figure 4. Cross-stream profiles of mean streamwise velocity at a number of streamwise locations. a) unforced; b) forced at the natural frequency (6 Hz); c) forced at one-half the natural frequency (3 Hz.).

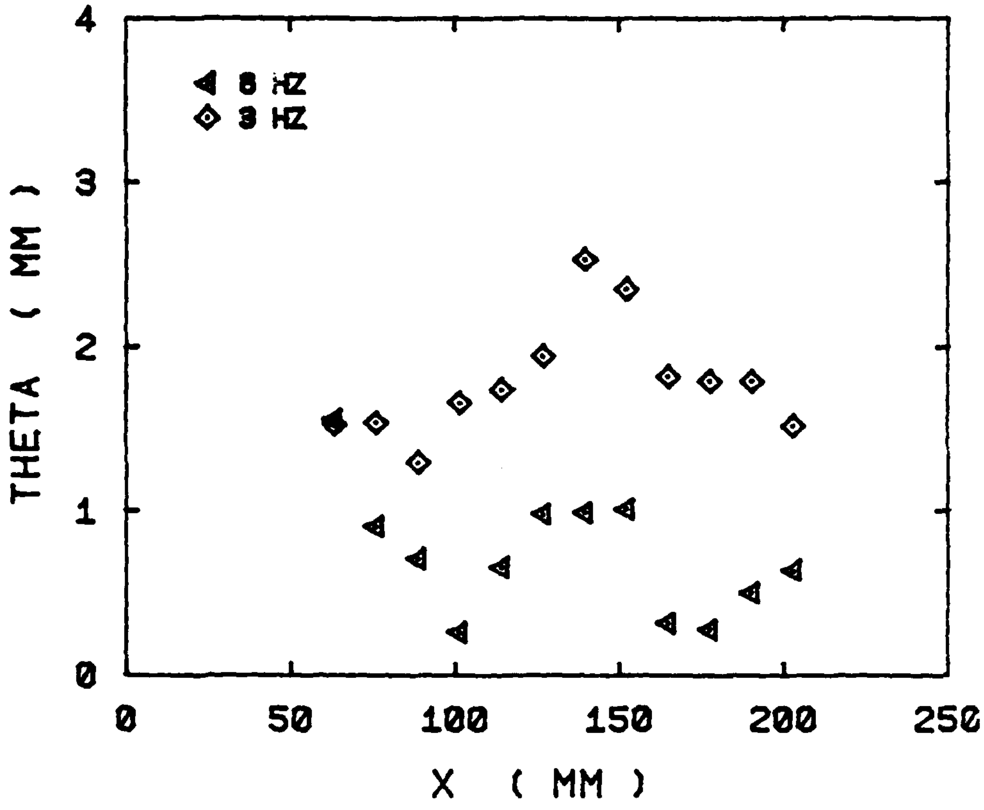


Figure 5. Streamwise difference in momentum thickness, $\Delta\Theta$, between the forced and unforced flow. a) at the natural frequency (6 Hz.); b) at one-half the natural frequency (3 Hz.).

A current focus of our work is the evolution of the streamwise vortices resulting from simultaneous excitation by a low-level spanwise-uniform harmonic wavetrain ($f_0 = 3.7$ Hz), and a spanwise-nonuniform steady waveform synthesized by an array of 16 heating elements, using heating mosaic I. Streaklines of fluid elements, slightly heated by the surface heaters on the flow partition, were visualized in the spanwise plane by means of a Schlieren system (§III.1.b), and recorded photographically. The streamwise evolution of various excitation waveforms between 10 cm and 25 cm downstream of the trailing edge of the flow partition ($U_1 = 18$ cm/sec, $U_2 = 6$ cm/sec) is shown in Figure 6. Even though these streaklines do not necessarily mark the presence of streamwise vorticity, they obviously suggest the formation of streamwise vortical structures, the spanwise extent of which varies significantly with the upstream excitation.

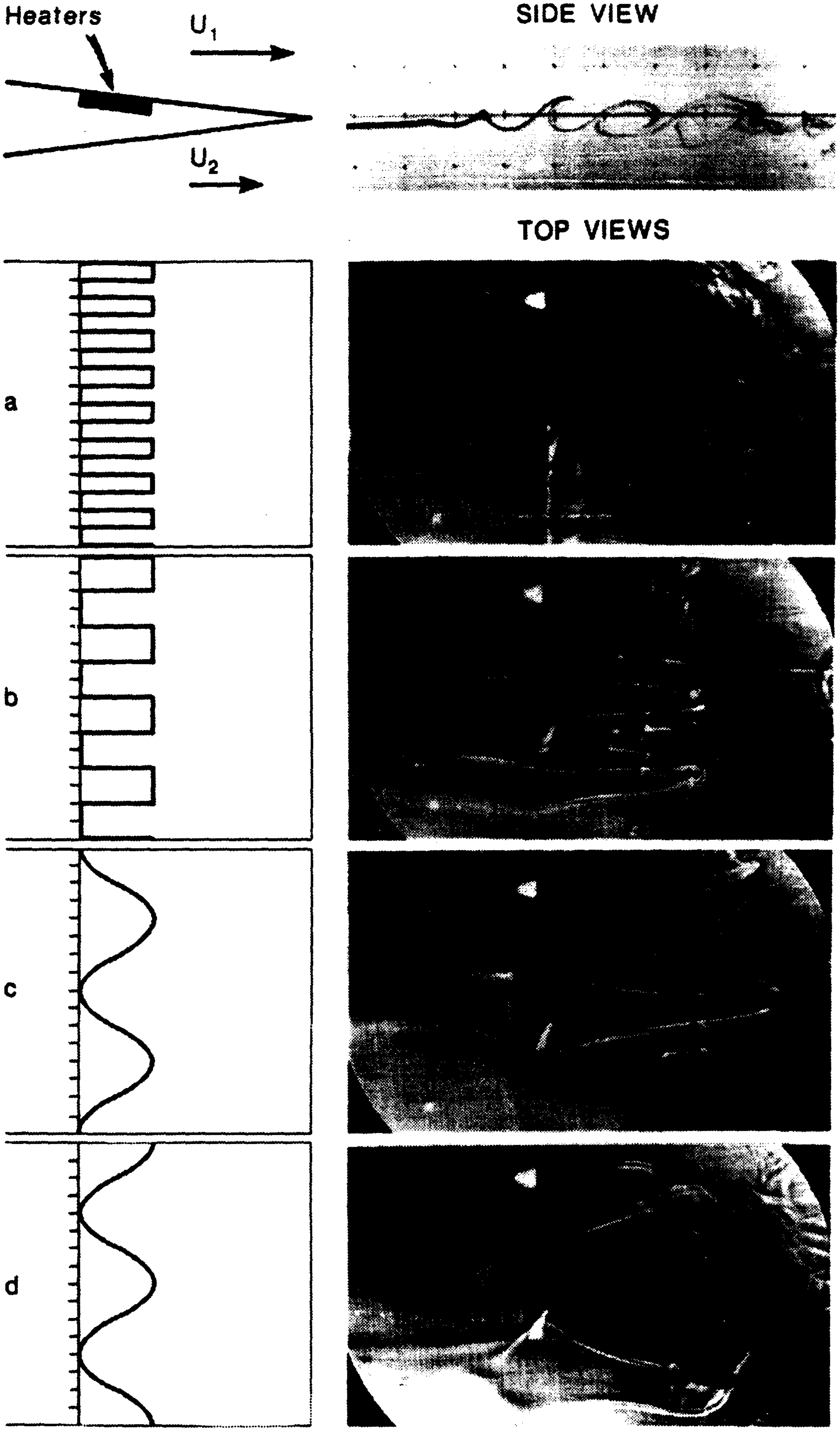
These investigations suggest that for a given spanwise-uniform excitation frequency, open-loop control can be effectively applied at <u>virtually any spanwise wavelength synthesizable by the spanwise heating array</u>. In order to enhance spatial resolution, heating mosaic I was used to synthesize a 16 element discretization of $1-\cos(\lambda z)$, where the spanwise wavelength is equal to the width of ten heating elements. In what follows, we describe a flow visualization study and discuss some of our measurements. The velocities of the two streams were 24.5 and 9 cm/sec.

During an excitation period of the spanwise-uniform wavetrain, the flow was illuminated in the spanwise plane by a strobe triggered at a phase delay relative to the zero crossings of the excitation signal and photographed using the Schlieren technique described in §III.1.b. The flow in the cross-stream plane was visualized by means of dye injection on the low-speed side and was photographed separately at the same phase relative to the zero crossings of the excitation signal. Figure 7 is a composite of eight pairs (a - h) of side- and span-views taken at equal time intervals during the excitation period. The field of view is between 2.5 and 12.4 cm downstream of the flow partition.

The streaklines in the span-views suggest the formation of hairpin eddy-like streamwise vortical structures the spanwise extent of which varies significantly with downstream distance. These streamwise vortices have their origin in a spanwise instability which results from continuous interaction between the 2-D instability and nonuniformities in the spanwise vorticity due to the spanwise excitation. The subsequent formation of spanwise vortices significantly alters the strain field within the shear layer and induces the appearance of spanwise-nonuniform concentrations of streamwise vorticity between adjacent 2-D vortices. These convected packets are continuously stretched and ingested by the spanwise vortices. Since entrainment is governed by the spanwise vortices, mixing transition is likely to occur when 3-D motion (at high enough Reynolds number) is

Figure 6. Response (10 cm $\leq x \leq$ 25 cm) to spanwise-nonuniform excitation, shown schematically to the left. Side view of the unforced flow (visualized by dye injection) is shown for reference.

Figure 7. Composite of side and plan views showing the streamwise evolution of spanwise excitation waveform "c" (Figure 6) at eight equal time intervals during a period of the 2-D wavetrain.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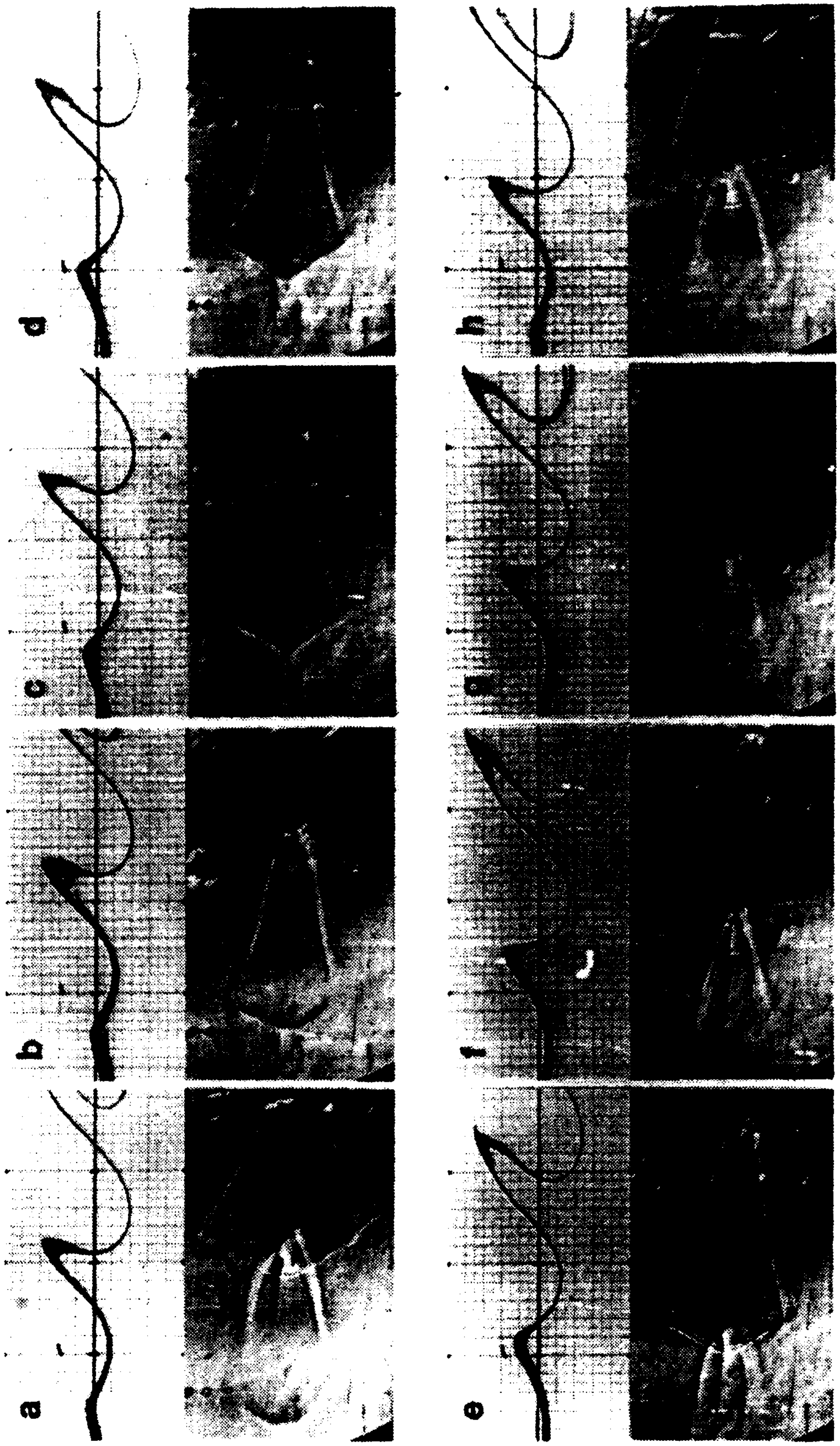
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established within their cores. We also conjecture that temporal evolution of the streamwise vortices in a shear layer excited by a spanwise-uniform wavetrain is locked to the 2-D instability modes.

Detailed measurements of the streamwise velocity phase-locked to the spanwise-uniform excitation have been made in (y-z) planes normal to the direction of the mean flow in two separate sets: with and without the spanwise-nonuniform steady waveform. Data were taken with a spanwise rake of 31 equally spaced (2 mm apart) hot-wire probes, suitable for use in water. The rake was traversed in the streamwise (x) and cross-stream (y) directions. The measurements presented below were made 15 cm downstream of the flow partition.

The changes in the mean velocity field which result from the spanwise nonuniform excitation can be educed, for example, from time-averaged cross-stream velocity profiles measured in the center of the span and shown in Figure 8 (o-with and o-without the spanwise-nonuniform steady waveform). We note that although there seems to be a rather small change in the cross-stream spreading of the flow, the momentum thickness increases by 40%.

While there is no question that the spanwise excitation alters the nominally 2-D structure of the flow, it is a nontrivial matter to extract a localized 3-D vortical structure from 3-D data of a single velocity component. Nevertheless, such a 3-D structure can be invaluable as a first step in understanding the dynamics of the flow. These vortical structures may be distinguished from the rest of the flow by the high intensity of the rms velocity fluctuations u'. A scheme by which u' is computed relative to each individual realization and then ensemble-averaged ($\langle\tilde{u}'\rangle$) has been developed and implemented. Unlike the conventional definition of the ensemble-averaged u', $\langle\tilde{u}'\rangle$ is not prone to spurious contributions from nonturbulent variations of the flow relative to its mean. The rms velocity fluctuation data, phase-averaged over the excitation period, were computed in the y-z plane, at x = 15 cm. Figures 9a and 9b show the surface $\langle\tilde{u}'\rangle = 0.085$ cm/sec in y-z-phase coordinates during two periods for the two spanwise-uniform excitation conditions considered here, namely with and without the spanwise-nonuniform steady waveform, respectively. Contours of $\langle\tilde{u}'\rangle$ in the y-z and y-phase planes are presented in Figure 10. Contours start at 0.05 cm/sec with intervals of 0.10 cm/sec; the respective phase and z coordinates of these planes are marked in Figure 9.

III.2.c. Two-dimensional Pulsed Excitation

As discussed in §II, most previous studies of forced (open-loop controlled) plane mixing layers have been restricted to simple time-harmonic excitation. Although the study of isolated

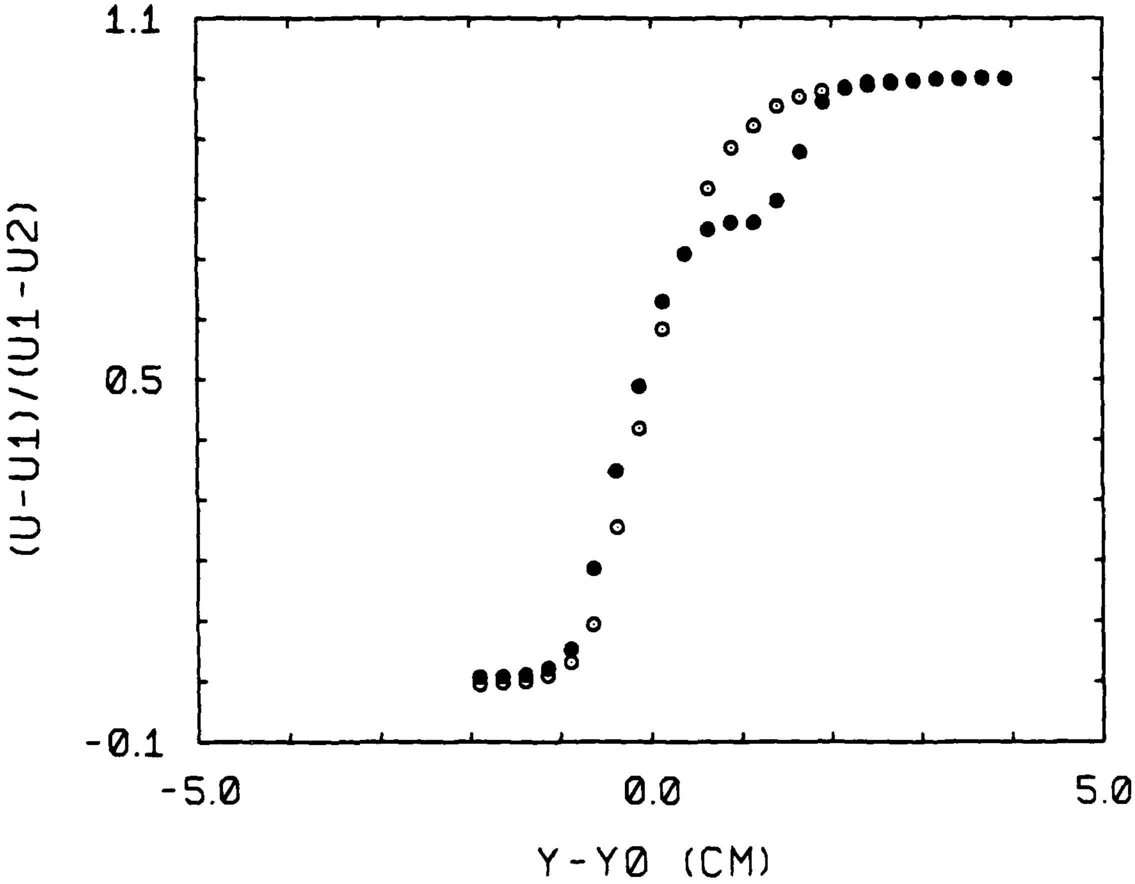


Figure 8. Time-averaged cross-stream velocity profiles measured in the center of the span at 150 mm downstream from flow partition (●-with and o-without the spanwise-nonuniform steady waveform).

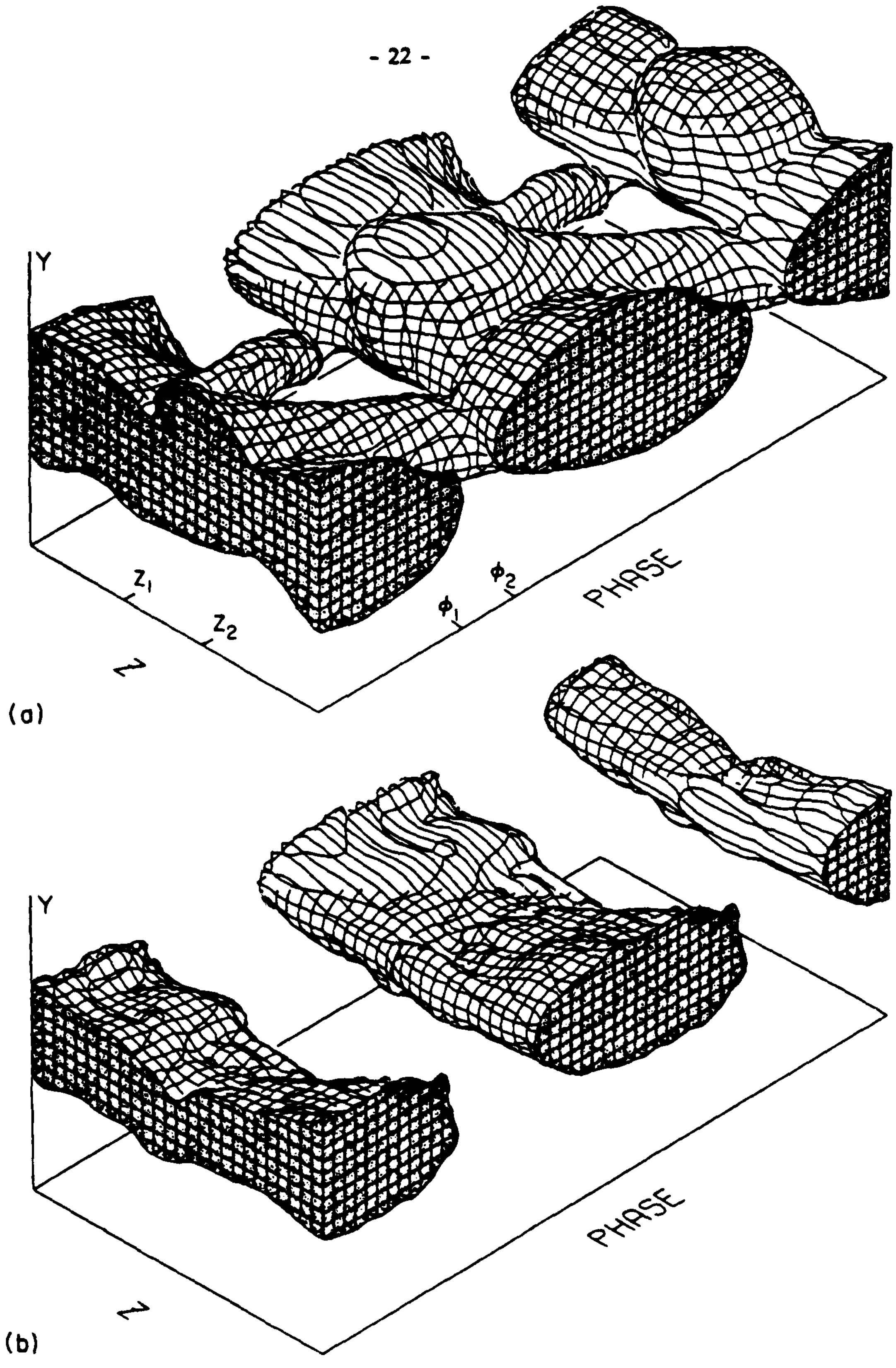
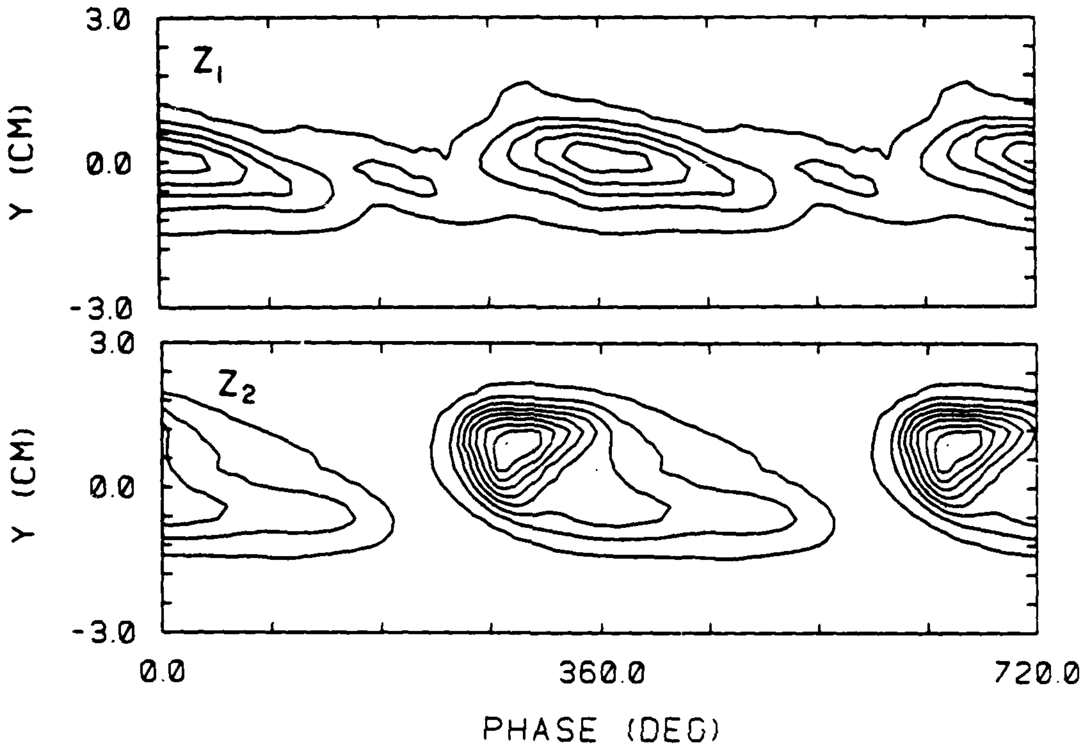


Figure 9. **Surfaces of constant rms velocity fluctuations (0.085 cm/sec) plotted in y-z-phase coordinates during two periods of the 2-D excitation with (a) and without (b) the the spanwise-nonuniform steady waveform.**



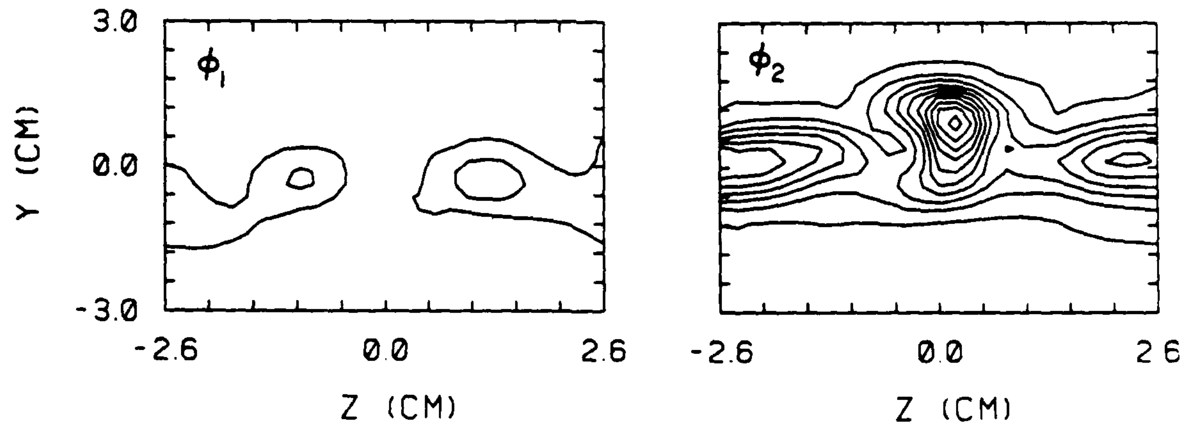


Figure 10. Contours of rms velocity fluctuations in the y-z and y-phase planes. Contour levels start at 0.05 cm/sec and the contour intervals are 0.10 cm/sec; the respective phase- and z-coordinates of these planes are marked in the figure.

instability modes, enhanced by harmonic excitation, has led to important insight into the evolution of the natural (unforced) flow, the temporal evolution of the latter could not be fully assesed. This is because within the domain of influence of the excitation (as may be measured, for example, by the rate of change of momentum thickness with downstream distance), the flow can evolve in space only while its temporal evolution is limited to the frequency of excitation and some of the leading harmonics. The importance of the problem in boundary layer transition was established by Gaster and Grant (1975) and later by Gaster (1987), who studied the evolution of a wave packet in a laminar boundary layer. They concluded that a packet may lead to transition much faster than a continuous wavetrain of larger amplitude.

In a recent investigation, Glezer, Wygnanski, and Gu (1988) have studied the spatial and temporal evolution of a pulsed disturbance in a plane mixing layer. The experiments were conducted in an open-return air shear layer facility; excitation was effected by motion of a flap mounted at the trailing edge of the flow partition and spanning the test section. The velocities of the two streams were $U_1 = 5$ m/sec and $U_2 = 3$ m/sec, the frequency of the harmonic excitation was 22 Hz and the tip amplitude of the flap motion a_f was either 0.5 or 0.75 mm. (The response of the plane mixing layer to excitation by a continuous wavetrain at 20 Hz and $a_f = 0.5$ mm was measured in the same facility and documented in detail by Gaster, Kit, and Wygnanski (1985) (their data set I).) These baseline conditions were chosen because the harmonically excited wavetrain is amplified throughout most of the available test section and becomes neutral approximately 850 mm downstream of the flap.

The open-return wind tunnel facility in which the data were obtained has been described in detail by Oster and Wygnanski (1982). The two air streams are driven by separate blowers into a test section 600 mm in the span, 500 mm high (normal to the two streams) and 2000 mm long. Measurements of the streamwise velocity component in a cross-stream ($z = 0$) plane were taken with a rake of ten hot-wire sensors nominally 1 cm apart. The rake was traversed in the cross-stream (y) direction to provide between 30 and 40 measurement points, 600mm. Data taken at the other streamwise locations reported in reference 13 were not considered in the present work.

A two-dimensional pulsed disturbance was generated by motion of a thin flap of 1 cm chord hinged to the trailing edge of the flow partition and driven by an electromagnetic actuator, the input to which (Figure 1) was given by

$$E = E_0 P \sin(\omega_0 t)$$

where

$$P = 1+p \quad \text{for } (n-1)T \leq t \leq nT$$

$$P = 1 \quad \text{for } nT \leq t \leq (n+k-1)T \ .$$

The data were obtained for $p = 2$ ($a_f = 0.5$ mm), $p = 4$ ($a_f = 0.75$ mm), and $k = 10$. One hundred data records, each containing at least two cycles of the continuous wavetrain preceding and following arrival of the disturbance, were digitized. In addition to the ten data channels, each record included the modulating signal. The data acquisition procedures and some of the basic processing techniques are reported by Glezer et al. (1988).

The response to the pulsed amplitude modulation is distinguishable from the response to the carrier signal alone and is felt across the entire width of the shear layer. Passage of the disturbance, which is advected at approximately the average velocity of the two streams (4 m/sec), causes a substantial temporal increase in the width of the shear layer. A demodulation technique was developed to discriminate between the response of the flow to the modulating pulse and the harmonic excitation. This technique decomposes the response of the flow into a family of "modal" two-dimensional wave packets. The decomposition enables one to study the features of the disturbance in detail, and, in particular, the propagation, amplification and some of the nonlinear aspects of its leading modal components. The "fundamental" packet is advected with the mean velocity of the two streams and its streamwise extent and dominant frequencies remain virtually unchanged with downstream distance.

The passage of the disturbance is accompanied by a spatial and temporal change in the momentum thickness of the harmonically excited flow. In connection with this change it was found that the cross-stream distributions of the velocity perturbation within the disturbance are similar to those of the harmonically excited flow at streamwise stations having the same momentum thickness thus implying similar stability features.

High turbulence levels, not prevalent in the harmonically excited shear layer, are detected within the disturbance and suggest the possibility of transient mixing enhancement.

More detailed description of the results is available in the enclosed manuscript by Glezer et al. (1988).

III.2.d. Amplitude Equations for Wave Packets in Slightly Inhomogeneous Unstable Flows

In an experimental paper, Glezer et al. (1988) study the evolution of two-dimensional wave packets in a mixing layer which are generated by modulations of the amplitude of the sinusoidal oscillating splitter plate. In the absence of these modulations, many of the important features of the large-scale structures can be described, with surprising and remarkable accuracy, by the linear instability modes of a (fictitious) base flow, whose velocity profile is the long-time average velocity. This remark has been confirmed by several independent studies in a variety of shear flows [Gaster et al., 1985 (in mixing layers); Wygnanski et al., 1986 (in wakes); Petersen and Samet, 1987 (in jets)]. The excitation of turbulent flows is desirable from the point of view of controlling the dynamics of the flow via the introduction of controlled large-scale structures; for example, the size of the separation zone on the suction side of an airfoil may be reduced by irradiating this zone with sound (Wygnanski, private communication).

One important finding of Glezer et al. (1988) was that high turbulence levels, not prevelent in the purely sinusoidally excited shear layer, were detached within the wave packet and this observation suggested the possibility of mixing enhancement *via* wave packets and amplitude modulated disturbances. Very little fundamental work is available in the literature on this interesting and potentially important topic.

In addition, a satisfactory theoretical description of wave packets in diverging base has been lacking; part of the research supported by this grant addresses the problem of providing a description of these flows. A complete discussion and derivation of the equations are given by Balsa (1988); here, we simply summarize our results.

The study of wave packets in conservative wave systems (i.e., "stable base flows") is a well-developed and beautiful subject; two of the most profound contributions to this subject were made by Whitham (1965) and Hayes (1970).

In a seminal paper, Whitham defined wave action density and flux in terms of derivatives, of an averaged Lagrangian, with respect to frequency and wavenumber, and showed that wave action obeys a conservation law. The averaged Lagrangian is obtained by substituting into the actual Lagrangian density an elementary progressive wave action with slowly varying amplitude, A, wavenumber, $\mathbf{k}$, and frequency, ω, and then averaging the resultant equation over one oscillation (while ignoring any changes in these slowly varying quantities). If we now define a local (slowly varying) phase, ϕ, whose suitable time and space derivatives give the local frequency and wavenumber, then the original variational principle, now applied to the averaged Lagrangian (which

is written in terms of the derivatives of the phase), yields the conservation law for the wave action. This law arises from the Euler equation corresponding to variations in the phase. The enormous beauty of Whitham's approach is its power to deal simply with progressive waves in nonlinear (as well as linear) conservative systems (e.g., it enables one to naturally define dispersion for nonlinear systems).

In an equally important paper, Hayes (1970) proposed an alternative approach to the definition of wave action. In Whitham's (1965) theory, this entity arises from a variational principle applied to an averaged Lagrangian, while in Hayes' work, we find an absolute conservation law, valid in physical space, defined over a periodic one-parameter family of solutions. These two approaches differ for modal waves; Hayes' approach gives a conservation law in physical space, and the integral of this law over cross-space gives Whitham's results.

In spite of the power of the approaches of Whitham (1965) and Hayes (1970), they are seldom applied to instability waves because the averaging (over the phase or over the one-parameter family of solutions) cannot be carried out since instability waves are not perfectly periodic (i.e., they have an exponential growth in addition to periodic oscillations). We succeeded in overcoming this difficulty by providing a complex Lagrangian for the development of a linear instability wave packet on diverging base flows and derived several versions of an "amplitude" equation.

The key result is that there is a perfectly natural generalization of the (classical) wave action density and flux; these obey the following conservation law

$$\frac{\partial \mathcal{A}}{\partial t} + \nabla \cdot (\underset{\sim}{G} \mathcal{A}) = \sigma \tag{1}$$

where $\mathcal{A}$ is the wave action density, $\underset{\sim}{G}$ is the complex group velocity and σ is the source term. Of course, the wave action flux is $(\underset{\sim}{G}\mathcal{A})$; the definitions of σ and $\mathcal{A}$ may be found in the paper by Balsa (1988). When the base flow is inviscid (i.e., a conservative system) the source term, σ, vanishes.

Very roughly speaking, (1) is a generalization of the amplitude equation of Crighton and Gaster (1975) when the disturbance consists of a spectrum of frequencies (or wavelengths). It is the wave action, $\mathcal{A}$, and not the complex amplitude, A, which is the important entity; the former is quadratic in the latter and is essentially the product of A^2 and the integral of the mode in cross-space.

An important problem, which must be addressed in future research, is the solution of (1) and some auxiliary equations for various types of base flows (say, mixing layers). This solution tells us how a wave packet (i.e., its frequency, wavenumber, amplitude, etc.) evolves in diverging flow; this proposed work represents a significant extension of the work of Gaster et al. (1985) to aperiodic (in time) excitation.

IV. PUBLICATIONS ACKNOWLEDGING THIS AFOSR GRANT

Balsa, T.F., "Amplitude Equations for Wave Packets in Slightly Inhomogeneous Unstable Flows," accepted for publication in J. Fluid Mech., 1988.

Fiedler, H.E., Glezer, A., and Wygnanski, I.J., "The Control of the Plane Mixing Layer - Some Novel Experiments", In AIAA Series on Progress in Astronautics and Aeronautics, Current Trends in Turbulent Research (Branover, H., Mond, M., and Ungar, Y., ed.), 112, pp. 30-64, 1988.

Glezer, A., Wygnanski, I.J., and Gu, X., "Amplitude Modulated Excitation of a Turbulent Mixing Layer", accepted for publication, Phys. Fluids, 1988.

Nygaard, K.J., and Glezer, A., "A Spanwise Nonuniformly Forced Plane Mixing Layer", to appear in the Proceedings of the Second European Turbulence Conference, Berlin: Springer, 1989.

Nygaard, K.J. "Construction, Instrumentation, and Testing of a Spanwise Forced Mixing Layer Facility", MS Thesis, University of Arizona, 1987.

Abstracts/Presentation

Gu, X., A. Glezer, and I. J. Wygnanski, "Spatial and Temporal Evolution of Momentary Disturbances in an Excited Turbulent Mixing Layer," presented at APS/DFD Meeting, Columbus, Ohio, November 1986.

Gu, X., A. Glezer and I.J. Wygnanski, "Evolution of a Pulsed Two-Dimensional Disturbance Superimposed on an Excited Turbulent Plane Mixing Layer", presented at the APS/DFD Meeting, Eugene, Oregon, November 1987.

Nygaard, K.J., A. Glezer and I.J. Wygnanski, "Concurrent Streamwise and Spanwise Forcing of a Turbulent Mixing Layer", presented at the APS/DFD Meeting, Eugene, Oregon, November 1987.

Nygaard, K.J., and A. Glezer, "Experimental Investigation of a Spanwise Forced Mixing Layer", presented at the Second European Turbulence Conference, Berlin, August 1988.

Nygaard, K.J. and A. Glezer, "Spanwise Nonuniformly Forced Plane Mixing Layer", presented at the APS/DFD Meeting, Buffalo, New York, November 1988.

V. REFERENCES

Aref, H. and Flinchem, E.P. (1984) "Dynamics of a Vortex Filament in a Shear Flow", J. Fluid Mech., 148, 477-497.

Ashurst, W.T. and Meiburg, E. (1988) "Three-Dimensional Shear Layers via Vortex Dynamics", J. Fluid Mech., 189, 87-116.

Balsa, T.F. (1988) "Amplitude Equations for Wave Packets in Slightly Inhomogeneous Unstable Flows," accepted for publication in J. Fluid Mech.

Benney, D.J. (1961) "A Non-Linear Theory for Oscillations in a Parallel Flow", J. Fluid Mech., 10, 209-236.

Bernal, L.P. (1981) "The Coherent Structure of Turbulent Mixing Layers. I. Similarity of the Primary Vortex Structure. II. Secondary Streamwise Vortex Structure", Ph.D. Thesis, California Institute of Technology, Pasadena.

Bernal, L.P. and Roshko, A. (1986) "Streamwise Vortex Structure in Plane Mixing Layers", J. Fluid Mech., 170, 499-525.

Breidenthal, R.E. (1978) "A Chemically Reacting, Turbulent Shear Layer", Ph.D. Thesis, California Institute of Technology, Pasadena.

Browand, F.K. and Troutt, T.R. (1980) "A Note on Spanwise Structure in the Two-Dimensional Mixing Layer", J. Fluid Mech., 97, 771-781.

Brown, G.L. and Roshko, A. (1974) "On Density Effects and Large Structure in Turbulent Mixing Layers", J. Fluid Mech., 64, 775-816.

Cohen, J. (1985) "Instabilities and Resonances in Turbulent Free Shear Flows", Ph.D. Dissertation, The University of Arizona, Tucson.

Corcos, G.M. and Lin, S.J. (1984) "The Mixing Layer: Deterministic Models of a Turbulent Flow. Part 2. The Origin of the Three-Dimensional Motion", J. Fluid Mech., 139, 67-95.

Crighton, D.G. and Gaster, M. (1976) "Stability of Slowly Diverging Jet Flow," J. Fluid Mech., 77, 397-413.

Decker, R. and Schafer, C.F. (eds.) (1988) "Mixing and Demixing Processes in Multiphase Flows with Application to Propulsion Systems", NASA Conference Publication 3006.

Didden, N., (1977) "Untersuchung Laminarer, Instabiler Ringwirbel mittels Laser-Doppler-Anemometrie", Diss., Univ. Göttingen; also Mitt. MPI und AVA, Göttingen, Nr. 64.

Fiedler, H.E., Nottmeyer, K., Wegener, P.P., and Raghu, S. (1985) "Schlieren Photography of Water Flow", Experiments in Fluids, 3, 145-151.

Gaster, M. (1987) Private Communication.

Gaster, M., Kit, E., and Wygnanski, I.J. (1985) "Large-Scale Structures in a Forced Turbulent Mixing Layer", J. Fluid Mech., 150, 23-39.

Gaster, M., Grant, I. (1975) "An Experimental Investigation of the Formation and Development of a Wave Packet in a Laminar Boundary Layer", Proc. R. Soc. Lond. A, 347, 253-269.

Glezer, A., Wygnanski, I.J. and Gu, X. (1988) "Amplitude Modulated Excitation of a Turbulent Mixing Layer", accepted for publication in Phys. Fluids.

Glezer, A. (1988) "On the Formation of Vortex Rings", accepted for publication in Phys. Fluids.

Glezer, A., Coles, D.E. (1987) "An Experimental Study of a Turbulent Vortex Ring", submitted to J. Fluid Mech.

Hayes, W.D. (1970) "Conservation of Action and Modal Wave Action," Proc. Roy. Soc. A, 320. 187-208.

Ho, C.-M., Huerre, P. (1984) "Perturbed Free Shear Layers", Ann. Rev. Fluid Mech. 16, 365-424.

Ho, C.-M., Huang, L.-S. (1982) "Subharmonic and Vortex Merging In Mixing Layers", J. Fluid Mech., 119, 443-473.

Jimenez, J. (1983) "A Spanwise Structure in the Plane Shear Layer", J. Fluid Mech., 132, 319-336.

Konrad, J.H. (1976) "An Experimental Investigation of Mixing in Two-Dimensional Turbulent Shear Flows with Applications to Diffusion Limited Chemical Reactions", Ph.D. Thesis, California Institute of Technology, Pasadena.

Lasheras, J.C., Cho, J.S. and Maxworthy, T. (1986) "On the Origin and Evolution of Streamwise Vortical Structures in a Plane, Free Shear Layer", J. Fluid Mech., 172, 231-258.

Lasheras, J.C. and Choi, H. (1988) "Three-Dimensional Instability of a Plane Free Shear Layer: An Experimental Study of the Formation and Evolution of Streamwise Vortices", J. Fluid Mech., 189, 53-86.

Lin, S.J. and Corcos, G.M. (1984) "The Mixing Layer: Deterministic Models of a Turbulent Flow. Part 3. The Effect of Plane Strain on the Dynamics of Streamwise Vortices", J. Fluid Mech., 141, 139-178.

Metcalfe, R.W., Orszag, S.A., Brachet, M.E., Menon, S., and Riley, J.J. (1987) "Secondary Instability of a Temporally Growing Mixing Layer", J. Fluid Mech., 184, 207-243.

Miksad, R.W. (1972) "Experiments on the Nonlinear Stages of Free-Shear-Layer Transition", J. Fluid Mech., 56, 695-719.

Oppenheim, A.K. (1987) "Fluid Mechanical Control of Combustion", Plenary Lecture, 11th Canadian Congress of Applied Mechanics, Edmonton, Alberta, May 31-June 4.

Oster, D., Wygnanski, I. (1982) "The Forced Mixing Layer Between Parallel Streams", J. Fluid Mech., 123, 91-130.

Petersen, R.A. and Samet, M. (1988) "On the Preferred Mode of Jet Instability," J. Fluid Mech., 194, 194, 153-173.

Pierrehumbert, R.T. and Widnall, S.E. (1982) "The Two- and Three-Dimensional Instabilities of a Spatially Periodic Shear Layer", J. Fluid Mech., 114, 59-82.

Roberts, F.A. (1985) "Effects of Periodic Disturbance on Structure and Mixing in Turbulent Shear Layers and Wakes", Ph.D. Thesis, California Institute of Technology, Pasadena.

Roberts, F.A. and Roshko A. (1985) "Effects of Periodic Forcing on Mixing in Turbulent Shear Layers and Wakes", AIAA Shear Flow Control Conference, March 12-14, Boulder, Colorado.

Roshko, A. (1981) "The Plane Mixing Layer: Flow Visualization Results and Three-Dimensional Effects", in The Role of Coherent Structures in Modelling Turbulence and Mixing, Lecture Notes in Physics, (Jimenez, J., ed.), 136, 208-217. Berlin: Springer.

Whitham, G.B. (1965) "A General Approach to Linear and Non-Linear Dispersive Waves Using a Lagrangian," J. Fluid Mech., 22, 273-283.

Wygnanski, I., Champagne, F., and Marasli, B. (1986) "On the Large-Scale Structures in Two-Dimensional, Small-Deficit, Turbulent Wakes," J. Fluid Mech., 168, 31-71.